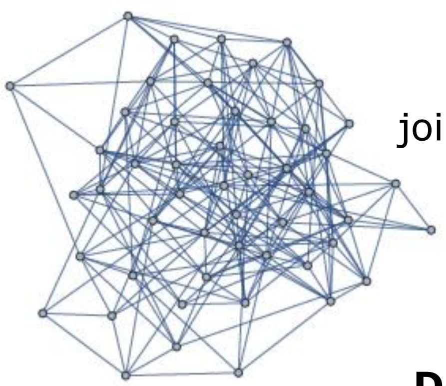
Analysis, design, and optimization of complex fractional dynamical networks



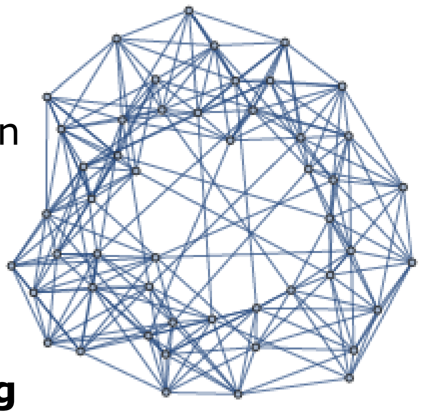
Sérgio Pequito

joint work w/ Qi Cao, Guilherme Ramos, and Paul Bogdan

goncas@rpi.edu

Department of Industrial and Systems Engineering

Rensselaer Polytechnic Institute



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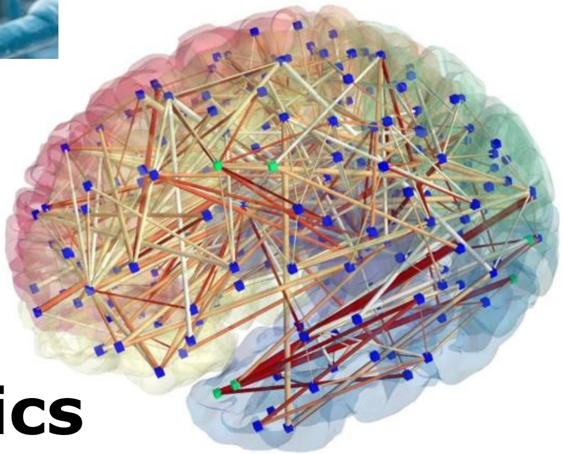
July 9th, 2019

ise

Complex Fractional Dynamical Networks

swarm robotics

bacteria dynamics

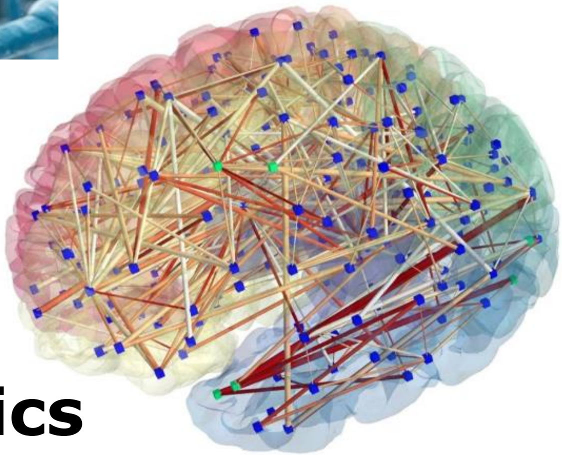


brain dynamics

Complex Fractional Dynamical Networks

swarm robotics

bacteria dynamics



brain dynamics

What do they have in common?

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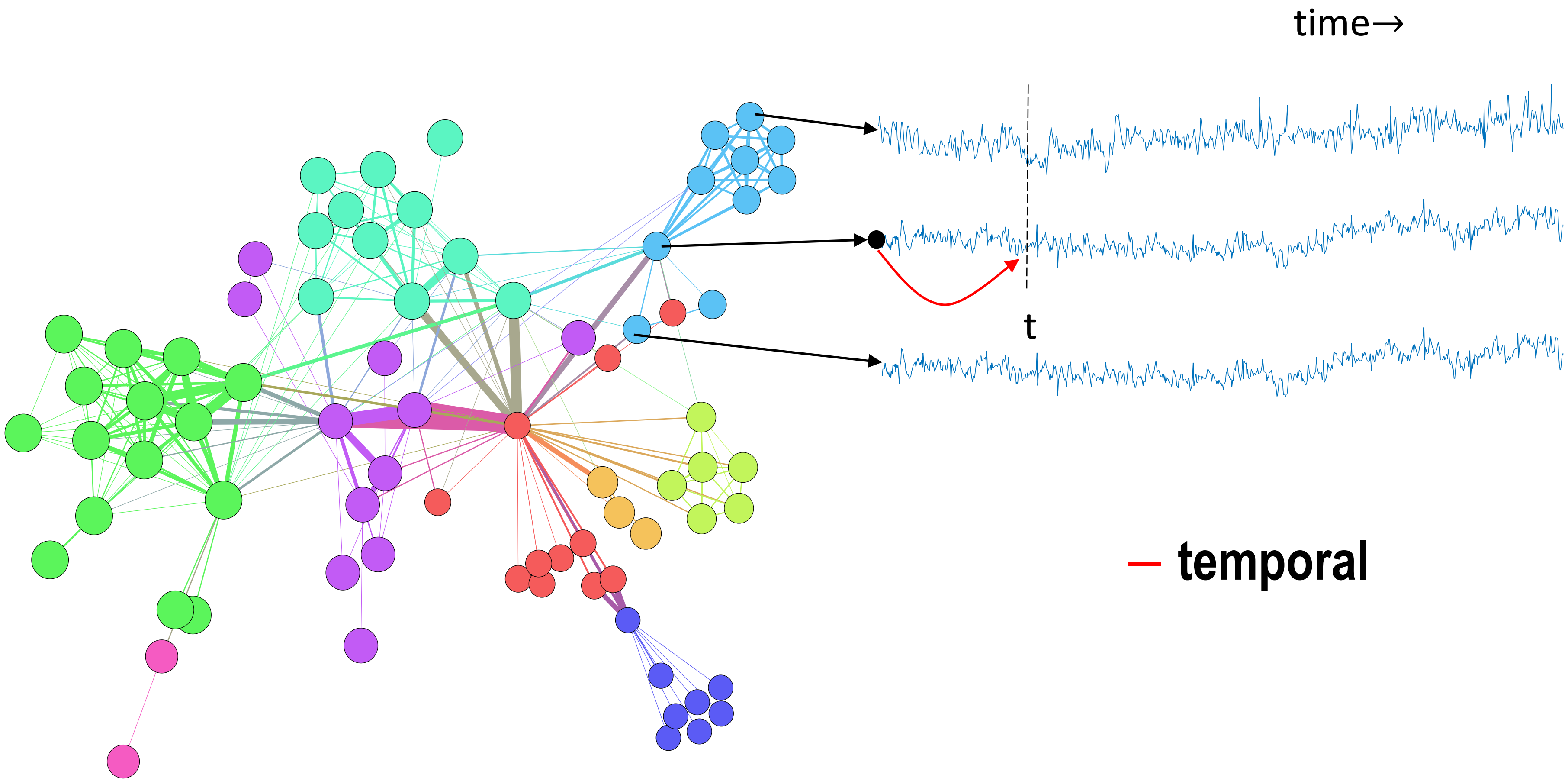
Complex Fractional Dynamical Networks

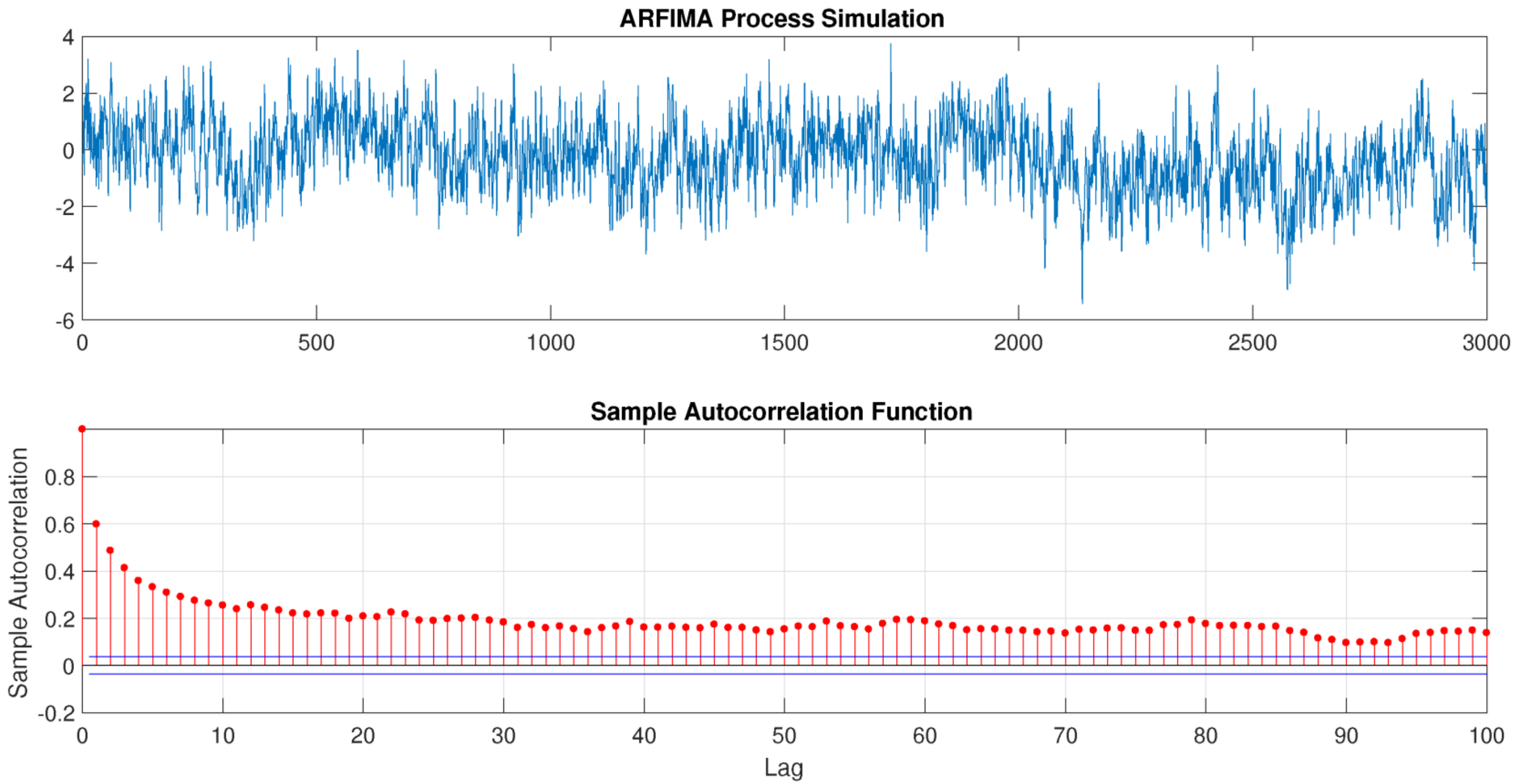


Complex Fractional Dynamical Networks

frontiers in
PHYSIOLOGY

METHODS ARTICLE
published: 04 June 2012
doi: 10.3389/fphys.2012.00141

Introduction to multifractal detrended fluctuation analysis in Matlab



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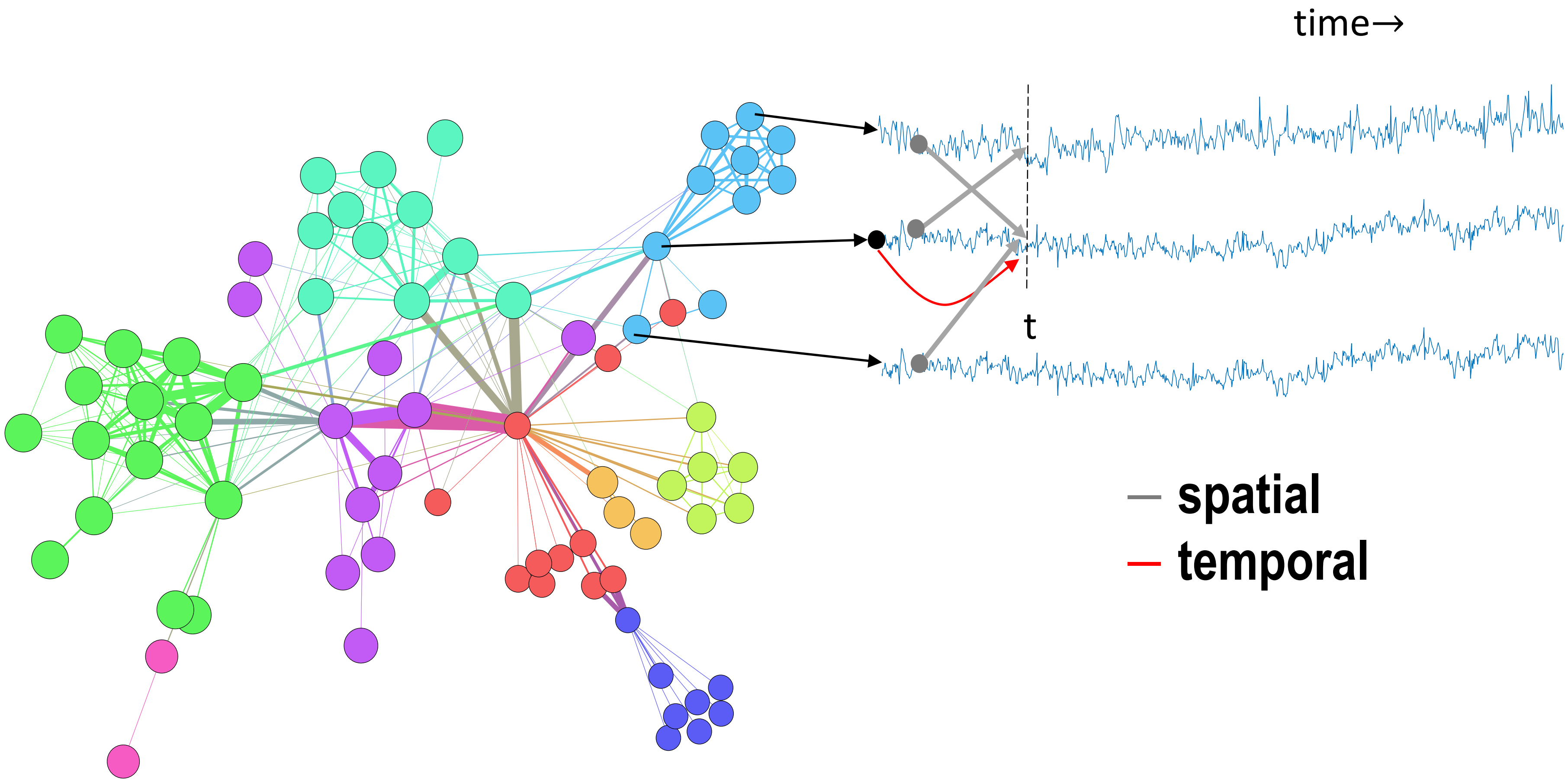
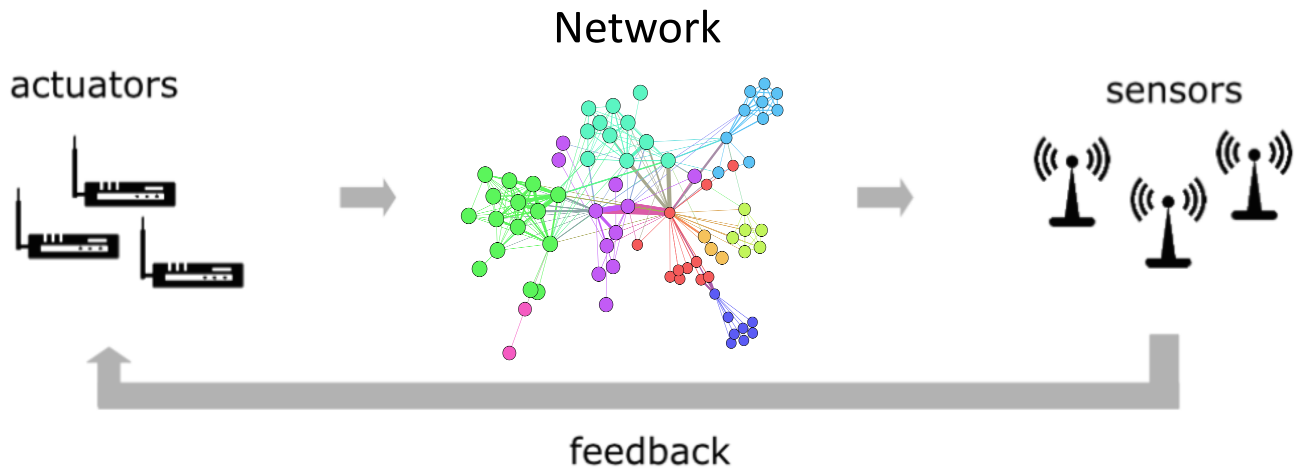
Complex Fractional Dynamical Networks

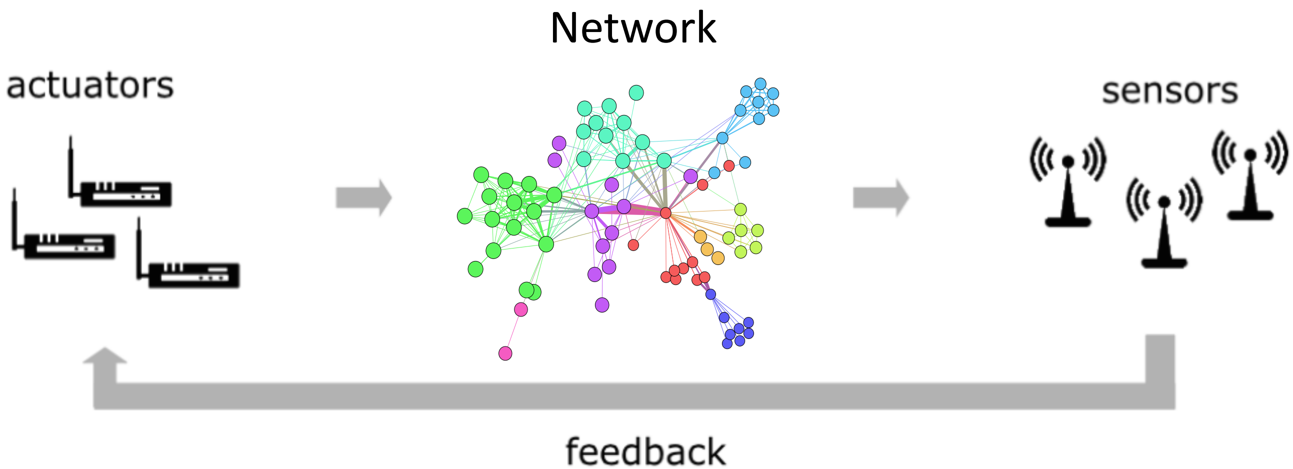


Today’s Outline



1. Crash course on Fractional “Stuff”
2. Minimum actuation requirements
3. Future Research and Open questions

Today's Outline



1. Crash course on Fractional "Stuff"
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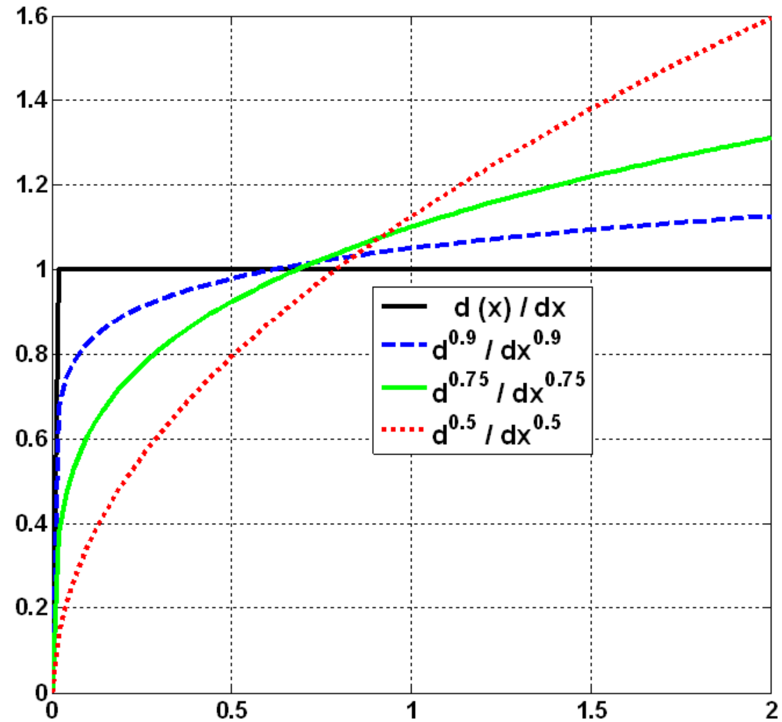
Fractional Calculus 101

We are all familiar with powers of a number

$$\frac{d}{dx}x^k, \frac{d^2}{dx^2}x^k, \frac{d^3}{dx^3}x^k, \ldots$$

What about powers (integer, and real) of an operator?

$$\frac{d^{\frac{1}{2}}}{dx^{\frac{1}{2}}}x^k, \frac{d^{\frac{1}{3}}}{dx^{\frac{1}{3}}}x^k, \frac{d^{\frac{1}{4}}}{dx^{\frac{1}{4}}}x^k, \ldots$$



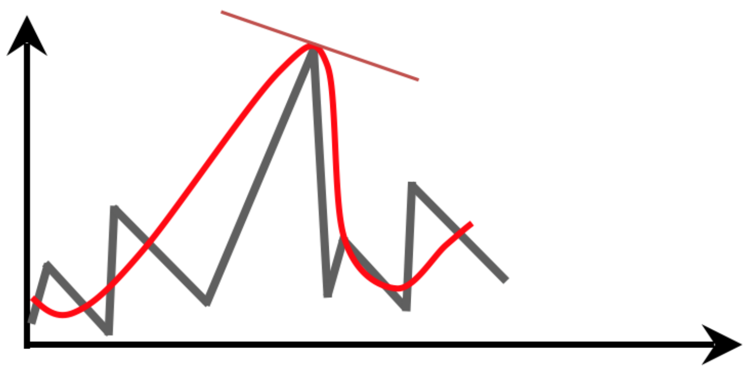
Fractional Order Derivative :

$$\frac{d^\alpha}{dx^\alpha}x^k = \frac{\Gamma(k+1)}{\Gamma(k-\alpha+1)}x^{k-\alpha}, \quad k \geq 0$$

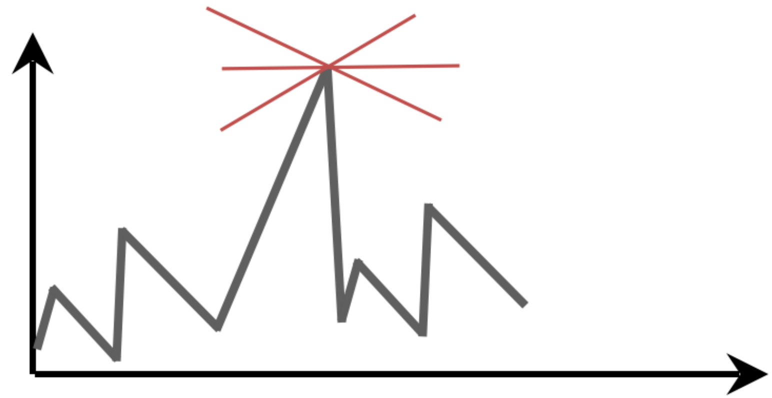
α is the fractal exponent

Fractional Calculus 101

Integer Calculus Approximation



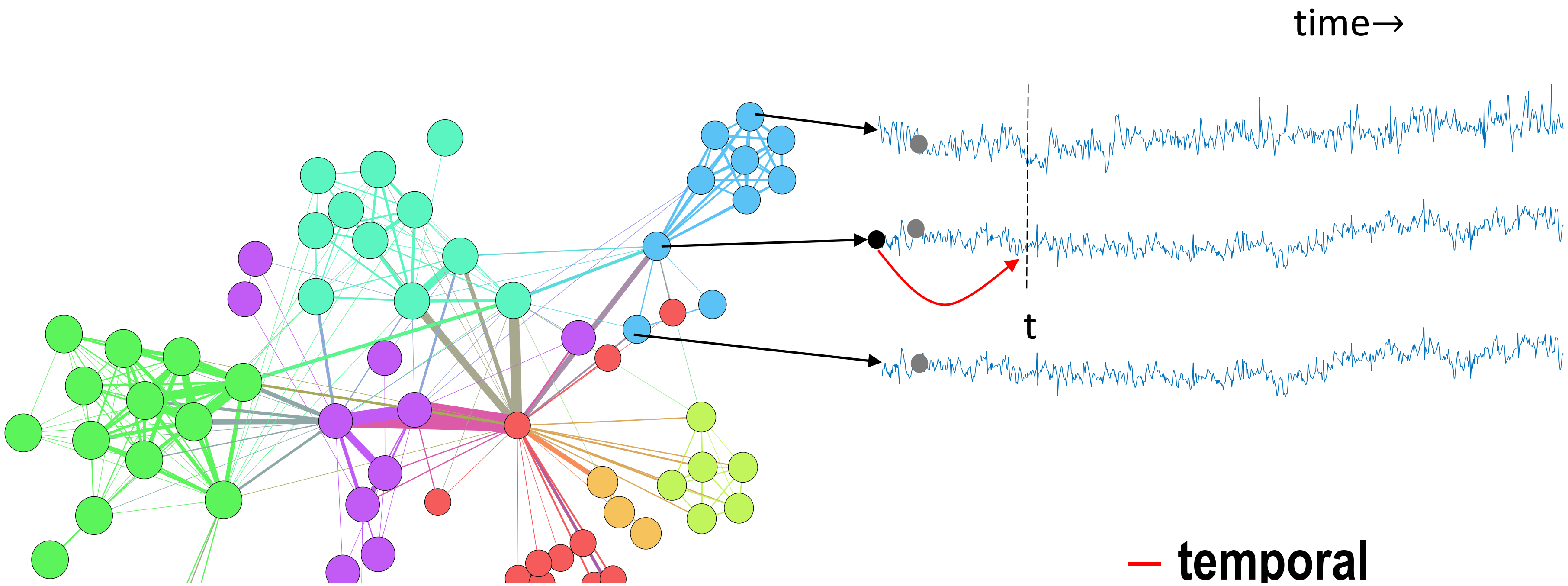
Fractal Calculus



$$\frac{d^{\alpha(t)}}{dt^{\alpha(t)}} y(t) = \lim_{\Delta t \to 0} \frac{1}{\Delta t^{\alpha(t)}} \sum_{j=0}^{\lfloor t/\Delta t \rfloor} (-1)^j \frac{\Gamma(\alpha(t)+1)}{\Gamma(j+1)\Gamma(\alpha(t)-j+1)} y(t - j\Delta t)$$

α is the fractal exponent

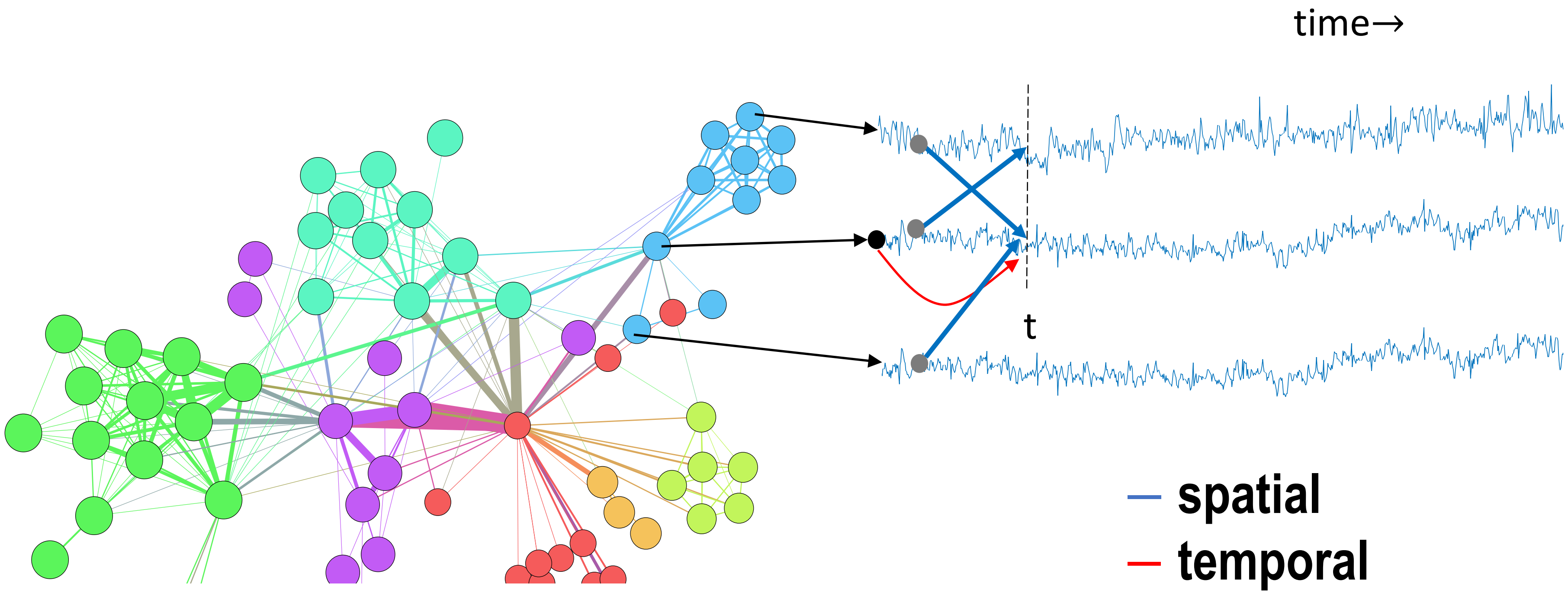
Complex Fractional Dynamical Networks



$$\begin{bmatrix} D^{\alpha_1} & & \\ & \ddots & \\ & & D^{\alpha_n} \end{bmatrix} x[k+1] = x[k], \quad k = 0, 1, \ldots, K$$

$$\alpha = (\alpha_1, \ldots, \alpha_n)$$

Complex Fractional Dynamical Networks



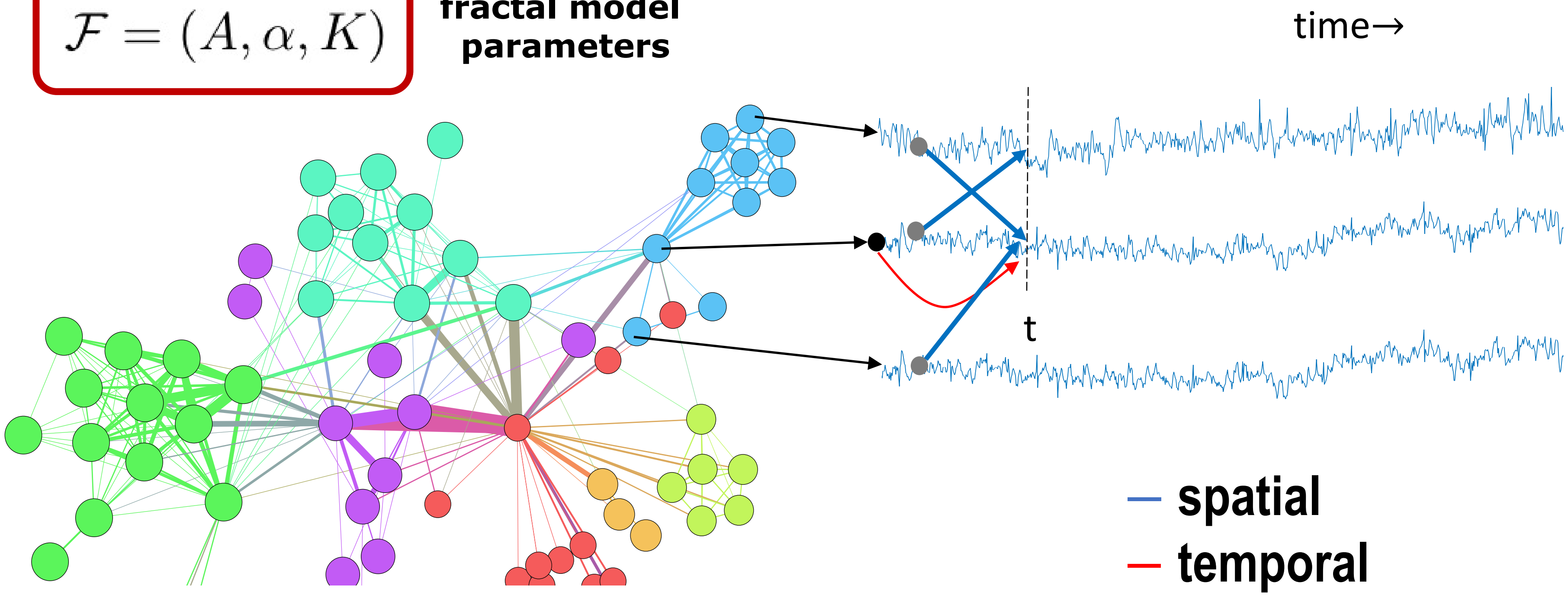
$$\begin{bmatrix} D^{\alpha_1} & & \\ & \ddots & \\ & & D^{\alpha_n} \end{bmatrix} x[k+1] = Ax[k], \quad k = 0, 1, \ldots, K$$

$$\alpha = (\alpha_1, \ldots, \alpha_n)$$

Complex Fractional Dynamical Networks

$$\mathcal{F} = (A, \alpha, K)$$

fractal model parameters



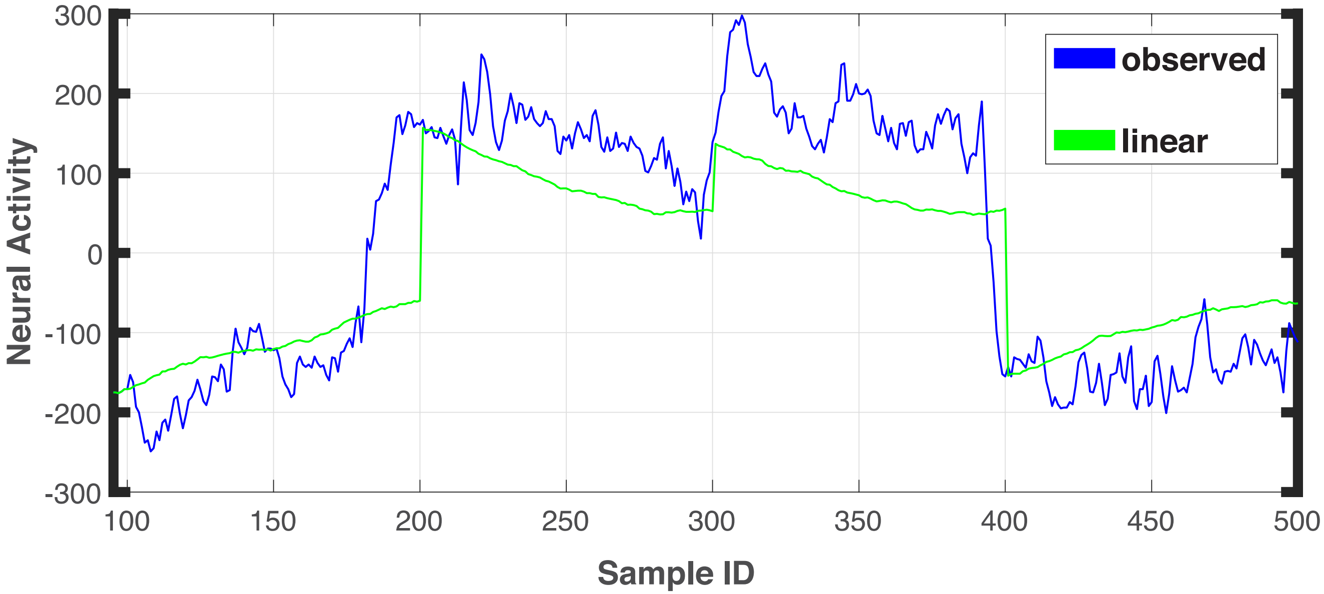
$$\begin{bmatrix} D^{\alpha_1} & & \\ & \ddots & \\ & & D^{\alpha_n} \end{bmatrix} x[k+1] = Ax[k], \quad k = 0, 1, \ldots, K$$

$$\alpha = (\alpha_1, \ldots, \alpha_n)$$

Complex Fractional Dynamical Networks

time series denoted as $x[k] \in \mathbb{R}^n$

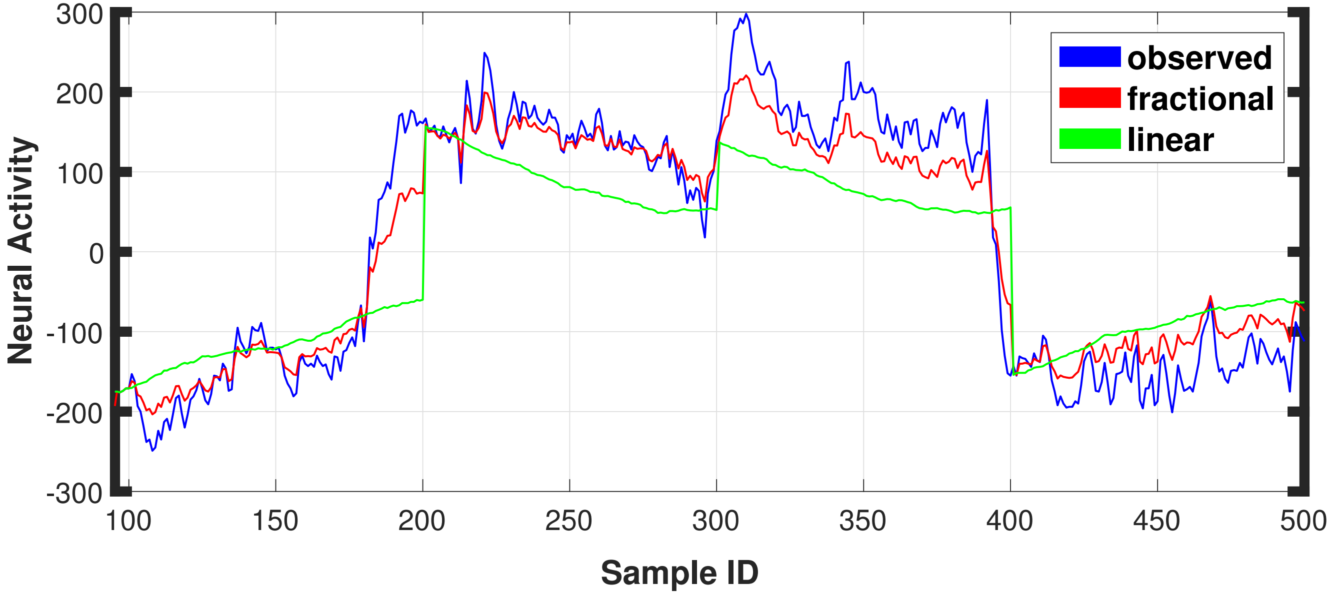
$$\boxed{\mathrm{D}^1 x[k+1] = Ax[k] + e[k]}$$



Complex Fractional Dynamical Networks

Fractional order model with spatial component

$$\boxed{\mathrm{D}^{\alpha}x[k+1] = Ax[k] + e[k]}$$



Control of Frac. Order Dyn. Net. (FODN)

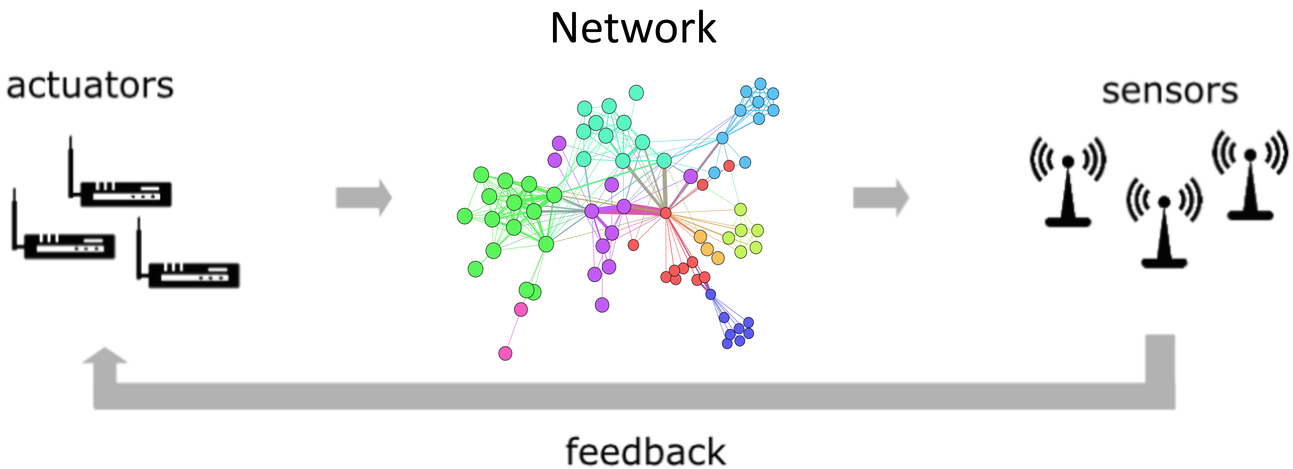
$$\mathcal{F} = (A, \alpha, K)$$

fractal model parameters

$$\begin{bmatrix} D^{\alpha_1} & & \\ & \ddots & \\ & & D^{\alpha_n} \end{bmatrix} x[k+1] = Ax[k] + Bu[k], \quad k = 0, 1, \ldots, K$$

The FODN described by $\mathcal{F}(A(\mathcal{G}), B; \boldsymbol{\alpha})$ is *controllable in K steps* if and only if, for every initial state $x_0 \in \mathbb{R}^n$ and desired final state $x_{\text{des}} \in \mathbb{R}^n$, there exists a sequence $u[0], \ldots, u[K-1]$ of inputs such that $x[0] = x_0$ and $x[K] = x_{\text{des}}$.

Today's Outline



1. Crash course on Fractional "Stuff"
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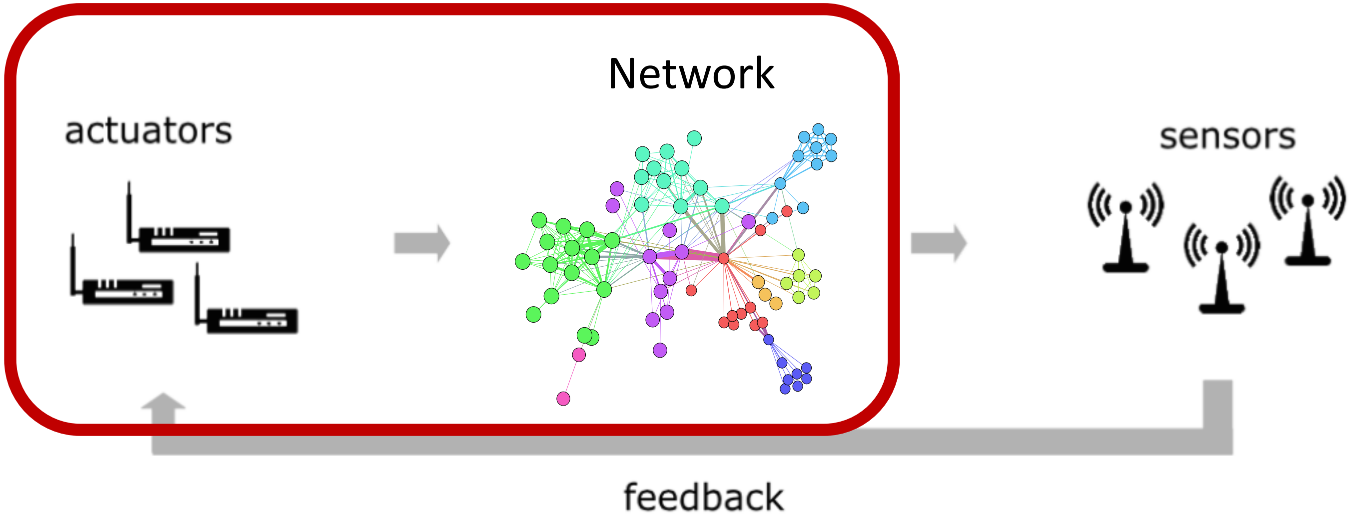
Minimum Actuation Requirements

The actuation spectrum for a FODN can be defined as a function $s : \mathbb{N} \to \{1, \ldots, n\}$ given by

$$s(K) \triangleq \min_{\mathcal{J} \subseteq \{1,\ldots,n\}} |\mathcal{J}|$$

$$\text{s.t.} \quad \mathcal{F}\big(A(\mathcal{G}), \mathbb{I}_n^{\mathcal{J}}; \boldsymbol{\alpha}\big) \text{ is controllable in } K \text{ steps.}$$



FODS "Behind the scenes"

Algebraic representation of the FODS as LTI switching system

$$x[k] = G_k x[0] + \sum_{j=0}^{k-1} G_{k-1-j} B u[j]$$

$$G_k = \begin{cases} \mathbb{I}_n & \text{for } k = 0 \\ \sum_{j=0}^{k-1} A_j G_{k-1-j} & \text{for } k \geq 1 \end{cases}$$

$$A_0 = A(\mathcal{G}) - \operatorname{diag}(\alpha_1, \ldots, \alpha_n) \text{ and } A_j = \begin{bmatrix} (-1)^j \binom{\alpha_1}{j+1} & & \\ & \ddots & \\ & & (-1)^j \binom{\alpha_n}{j+1} \end{bmatrix}$$

FODS "Behind the scenes"

Algebraic representation of the FODS as LTI switching system

$$x[k] = G_k x_0 + \underbrace{\left[G_0 B \ \ldots \ G_{k-1} B\right]}_{\mathcal{C}_k} \begin{bmatrix} u[k-1] \\ \vdots \\ u[0] \end{bmatrix}$$

$$\begin{bmatrix} u[K-1] \\ \vdots \\ u[0] \end{bmatrix} = (\mathcal{C}_K^\top \mathcal{C}_K)^{-1} \mathcal{C}_K^\top (x_{\text{des}} - G_K x_0)$$

$$\boxed{\mathcal{F}(A(\mathcal{G}), B; \boldsymbol{\alpha}) \text{ is controllable in } K \text{ steps if and only if } \operatorname{rank}(\mathcal{C}_K) = n}$$

Minimum Actuation Requirements

The actuation spectrum for a FODN can be defined as a function $s : \mathbb{N} \to \{1, \ldots, n\}$ given by

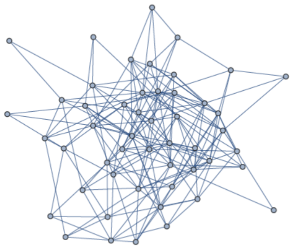
$$s(K) \triangleq \min_{\mathcal{J} \subseteq \{1,\ldots,n\}} |\mathcal{J}|$$
$$\text{s.t.} \quad \mathcal{F}\big(A(\mathcal{G}), \mathbb{I}_n^{\mathcal{J}}; \boldsymbol{\alpha}\big) \text{ is controllable in } K \text{ steps.}$$

NP-hard but submodular

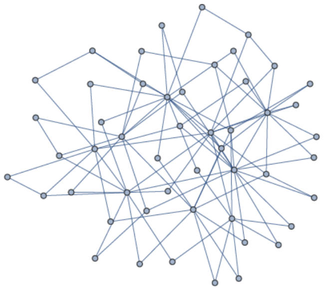
Apply greedy algorithms with optimality guarantees

Minimum Actuation Requirements

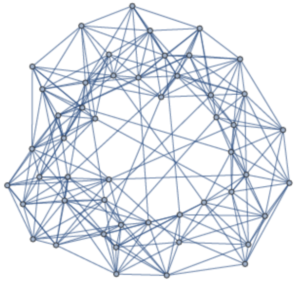
Artificial Complex Networks



Erdös-Rényi (ER) Network Model



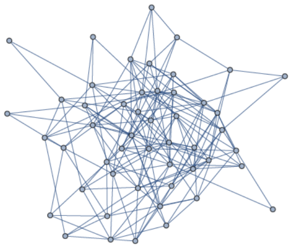
Barabási-Albert (BA) Network Model



Watts-Strogatz (WS) Network Model

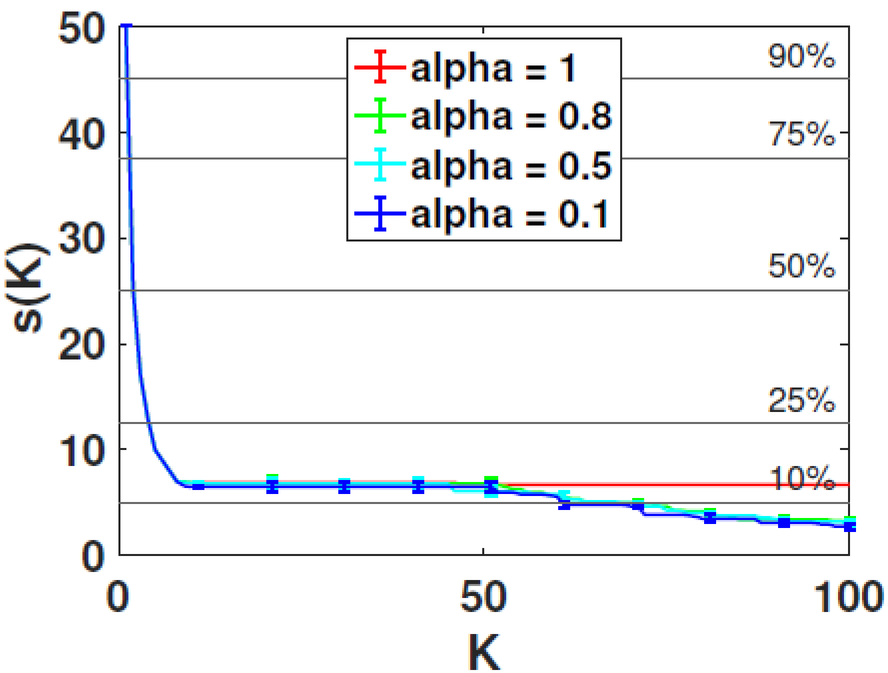
Minimum Actuation Requirements

n=50

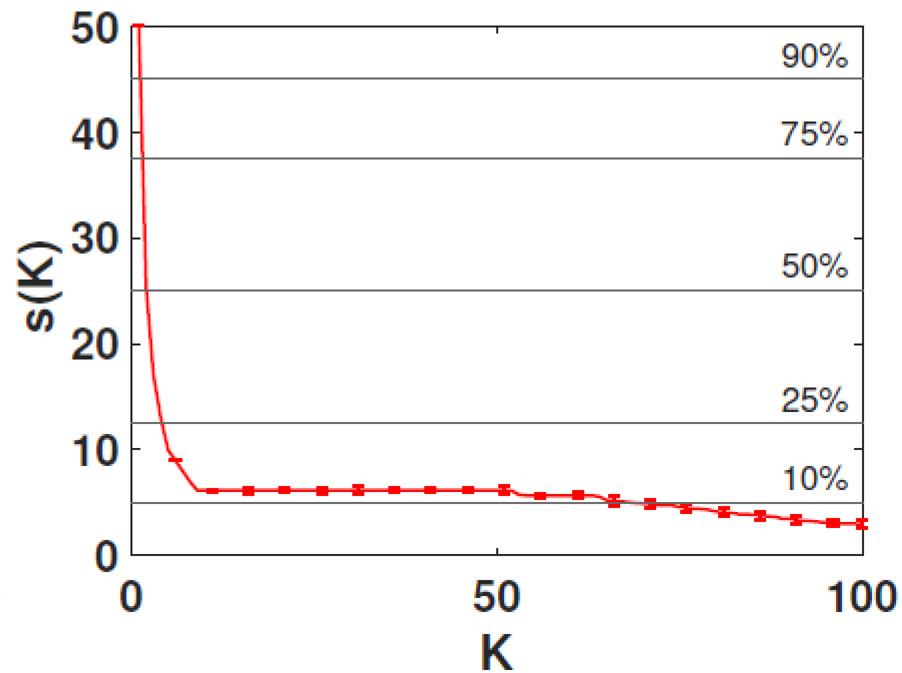


Erdös-Rényi (ER) Network Model

$p = 0.15$



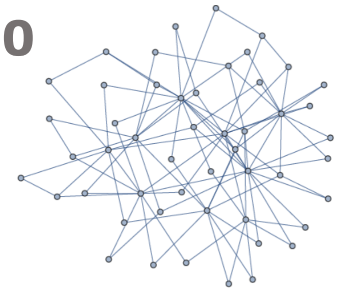
AS with fixed α



AS with $\alpha \sim \mathcal{U}(0,1)$

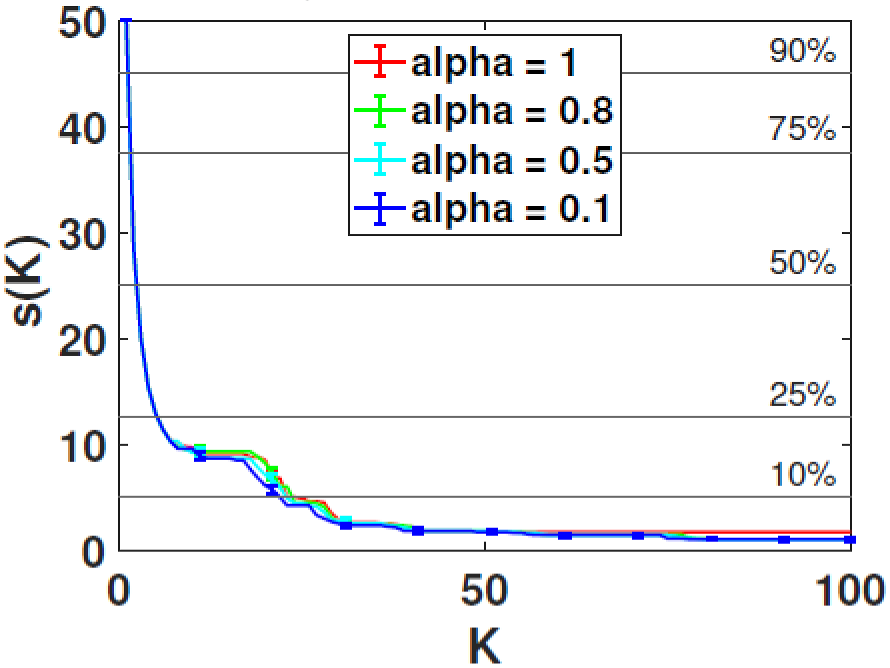
Minimum Actuation Requirements

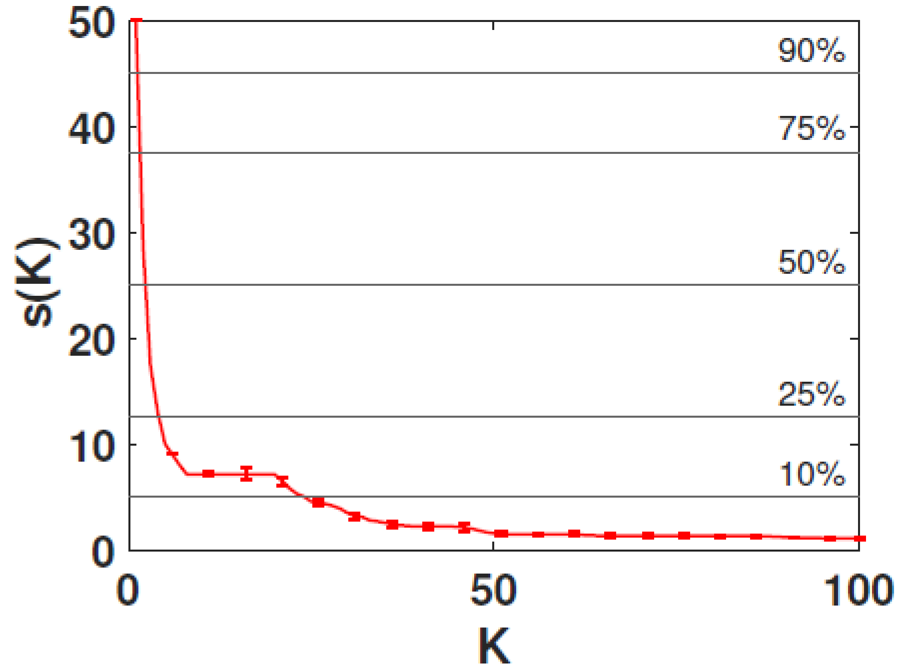
n=50



Barabási-Albert (BA) Network Model

$m = 2$



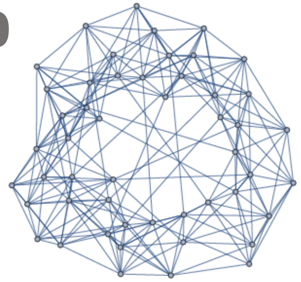


AS with fixed α

AS with $\alpha \sim \mathcal{U}(0,1)$

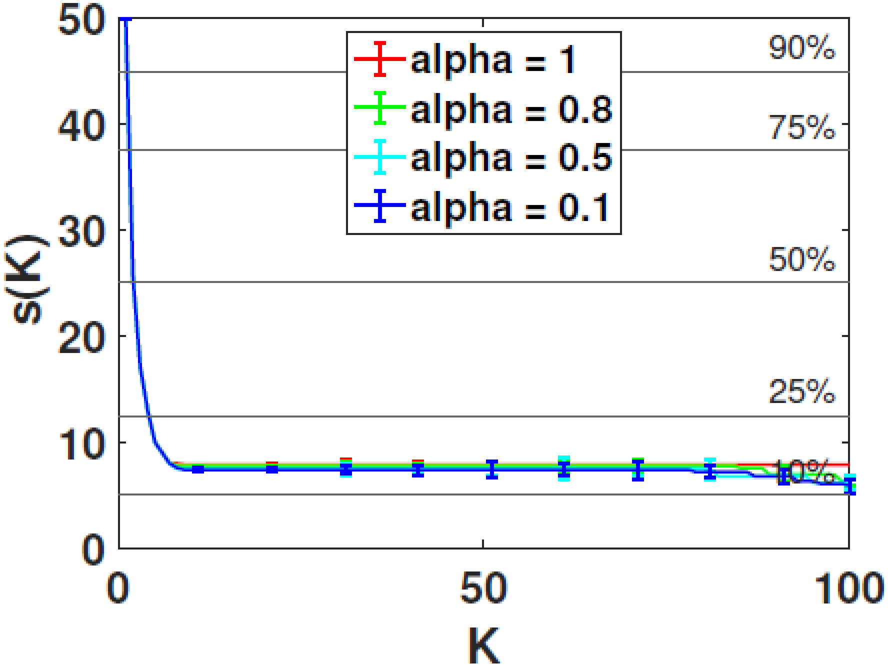
Minimum Actuation Requirements

n=50

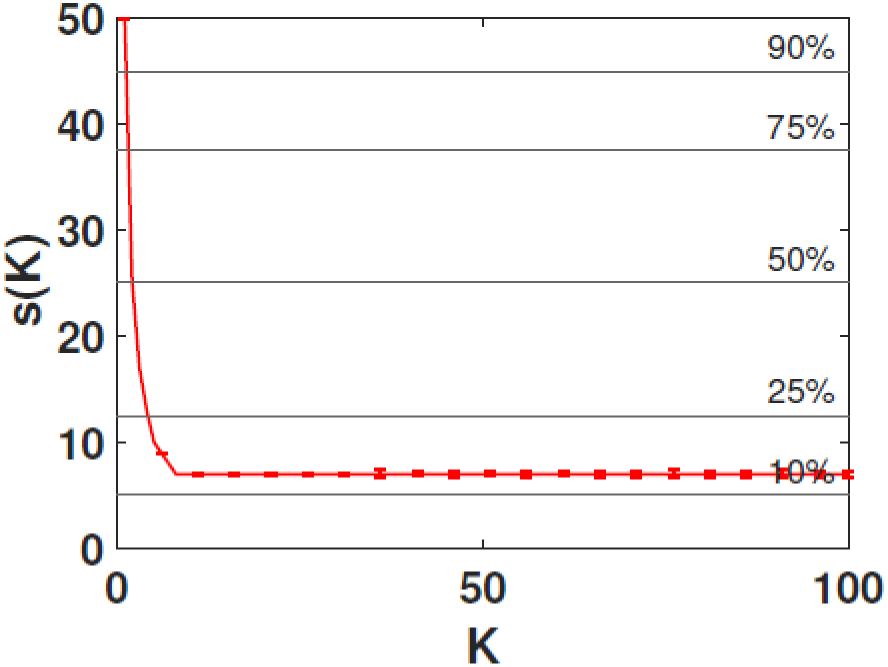


Watts-Strogatz (WS) Network Model

$$d = 10, \beta = 0.1$$



AS with fixed α



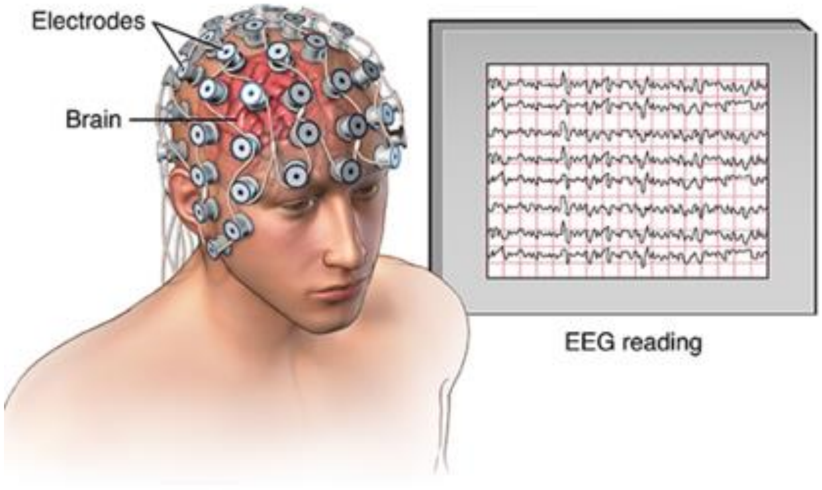
AS with $\alpha \sim \mathcal{U}(0,1)$

Minimum Actuation Requirements

Real Complex Network

Skull-level electroencephalographic (EEG) data in resting state [100 channels]

n=100



LTI

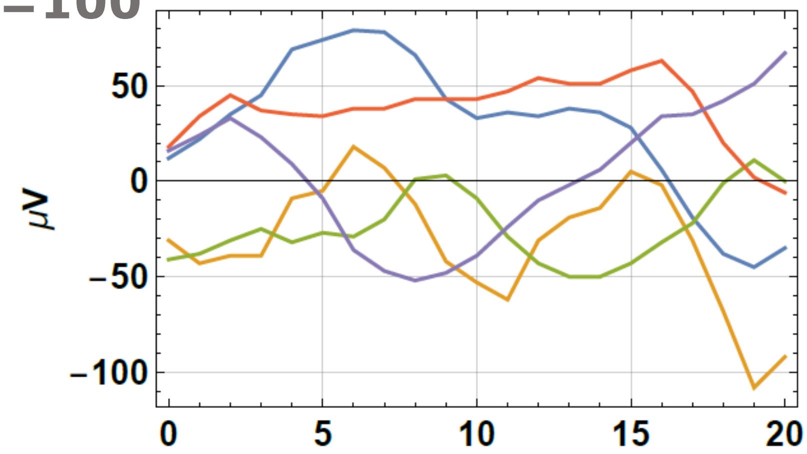
$$\mathrm{D}^1 x[k+1] = Ax[k] + e[k]$$

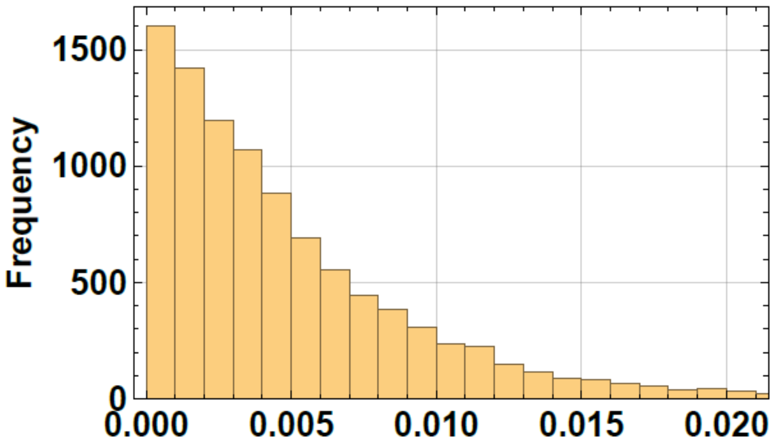
FODN

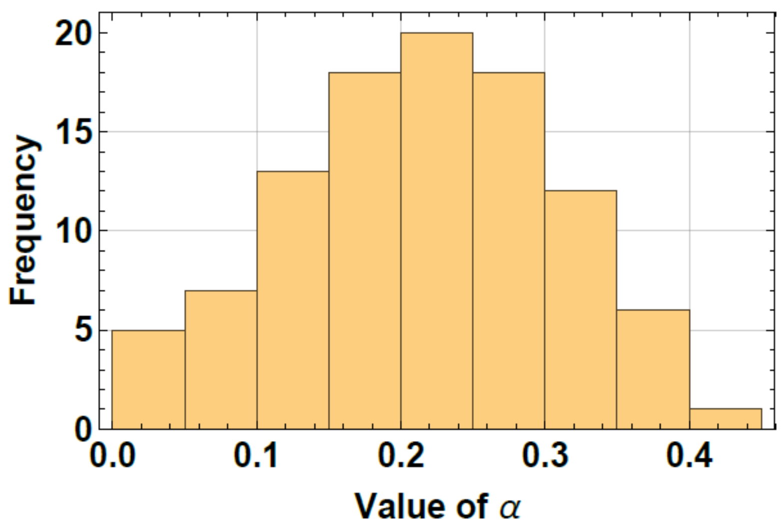
$$\mathrm{D}^\alpha x[k+1] = Ax[k] + e[k]$$

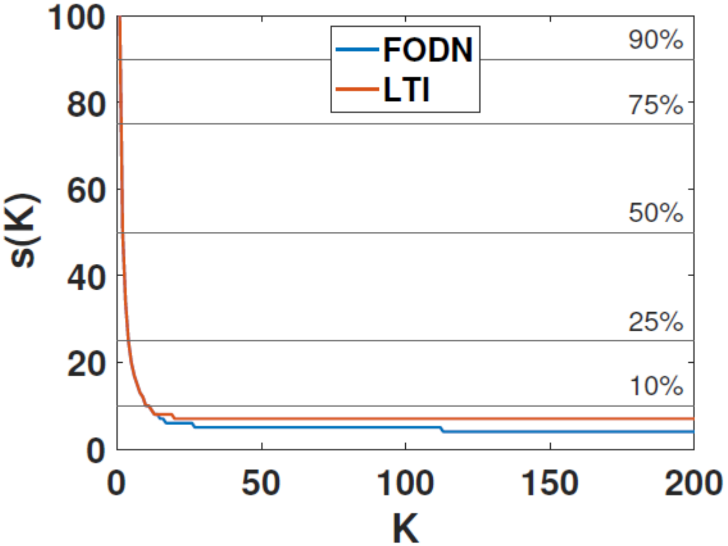
Minimum Actuation Requirements

n=100



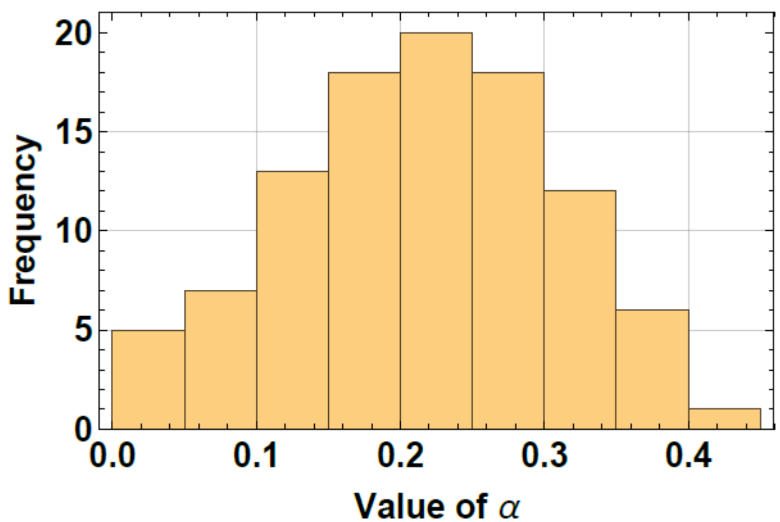


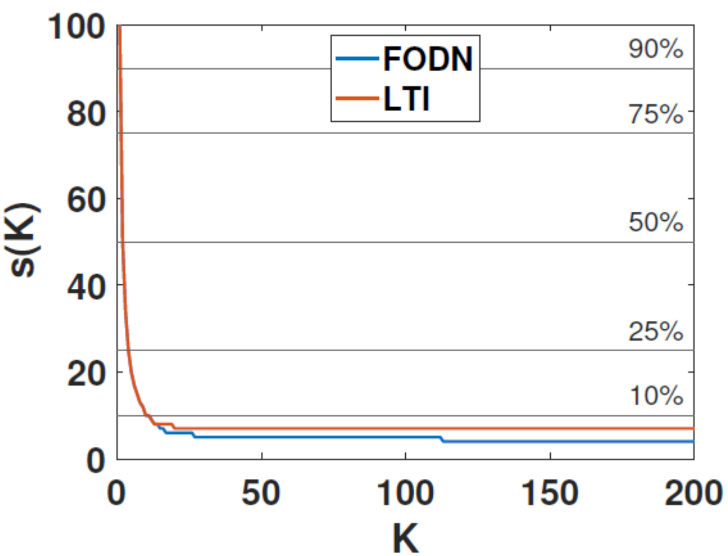




Minimum Actuation Requirements

We can use actuation capabilities to assess if the model is inherently LTI or FODN by considering the minimum actuation capabilities required for the FODN. In other words, if we can steer the system towards a desired state with the minimum actuation capabilities required for the FODN then it cannot be an LTI.





Minimum Actuation Requirements

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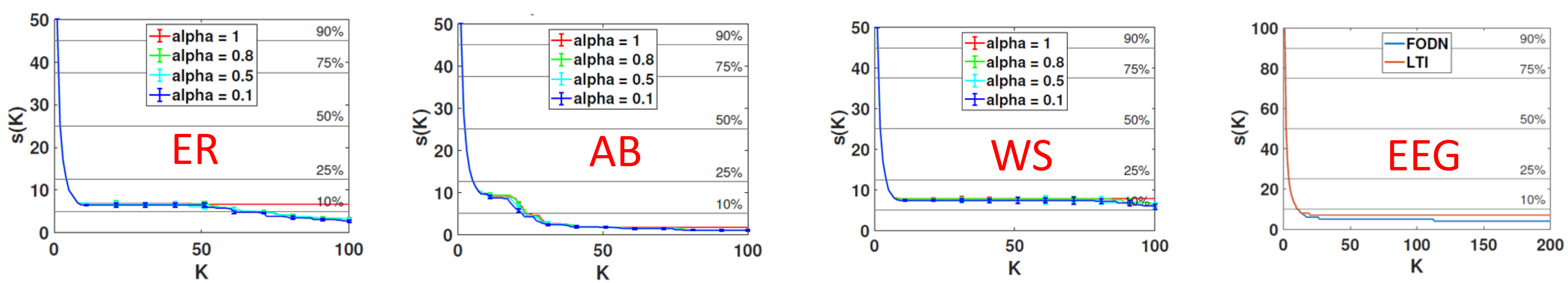
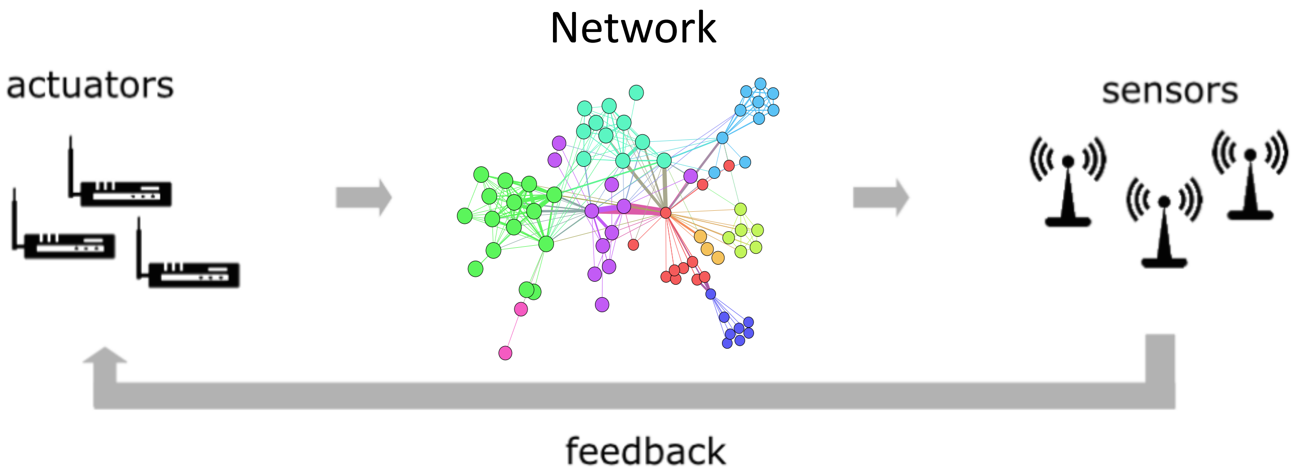
Notice that actuation spectra across artificial and real networks are different, which implies that we need generative models capable of capturing the actuation spectra of real networks



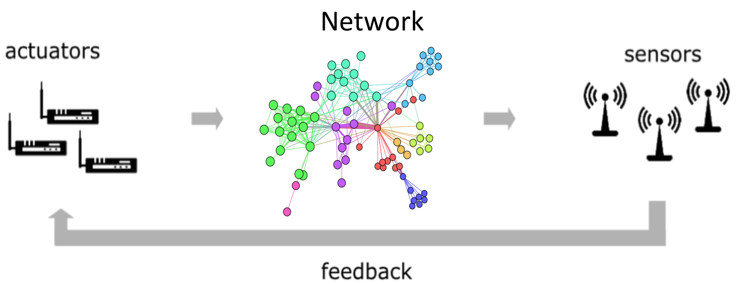
Today's Outline



1. Crash course on Fractional "Stuff"
2. Minimum actuation requirements
3. Research and Open questions

Research and Open questions

Fractional order dynamical networks are modeled by nonlinear systems parametrized by a small number of temporal and spatial parameter with a high description power. Although there is a strong analysis foundation for the numerical approximations schemes from its continuous counterpart it is missing a sound control theory.



The promise of a solid control theory lies in the fact that these are not arbitrary nonlinear systems (i.e., they exhibit a lot of structure).

<u>Problems that are required to be addressed:</u> *(i)* resilient-robust state estimation; *(ii)* computationally efficient system id with guarantees; *(iii)* robust control; *(iv)* decentralized control; and *(v)* MPC and its variants.

Research Bibliography

Qi Cao, Guilherme Ramos, Paul Bogdan, **S. Pequito**, *On the actuation spectrum of spatiotemporal networks with power-law time dependencies*, Advances in Complex Systems, under review

Recently, our research group and collaborators published the following papers:

Andrea Alessandretti, **S. Pequito**, George J. Pappas, A. Pedro Aguiar, *On the finite-dimentional control of discrete-time fractional order systems*, to appear in Automatica

Sarthak Chatterjee, Orlando Romero, and **S. Pequito**, *A Separation Principle for Discrete-Time Fractional-Order Dynamical Systems and its Implications to Closed-loop Neurotechnology*, IEEE Control Systems Letters, 2019

Sarthak Chatterjee and **S. Pequito**, *Dealing with State Estimation in Fractional-Order Systems under Artifacts*, Proceedings of the **2019 American Control Conference**

WeB06.6

Gaurav Gupta, **S. Pequito**, and Paul Bogdan. *Learning Latent Fractional dynamics with Unknown Unknowns*, Proceedings of the **2019 American Control Conference**

WeA06.6

Research Bibliography

- **S. Pequito**, P. Bogdan, and G. J. Pappas, Minimum Number of Probes for Brain Dynamics Observability, Proceedings of the IEEE 54th Conference on Decision and Control

- **S. Pequito**, Andrew Clark, and George J. Pappas, Discrete-time Fractional-order Multiple Scenario-based Sensor Selection, Proceedings of the 2017 American Control Conference

- Yuankun Xue, **S. Pequito**, Joana M. Coelho, Paul Bogdan, and George J. Pappas, Minimum Number of Sensors to Ensure Observability of Physiological Systems: a Case Study, Proceedings of the 54th Annual Allerton Conference on Communication, Control, and Computing

- V. Tzoumas, Y. Xue, **S. Pequito**, P. Bogdan, G.J. Pappas, "Selecting sensors in biological fractional-order systems ", IEEE Transactions on Control of Network Systems, 2018

- Gaurav Gupta, **S. Pequito**, and Paul Bogdan. "Dealing with Unknown Unknowns: Identification and Selection of Minimal Sensing for Fractional Dynamics with Unknown Inputs". Proceedings of the 2018 American Control Conference

- Gaurav Gupta, **S. Pequito**, and Paul Bogdan. "Re-thinking EEG-based non-invasive brain interfaces: modeling and analysis", Proceedings of the 9th ACM/IEEE International Conference on Cyber-Physical System, 2018

Thank you!