

Structural Controllability of Symmetric Networks

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Abstract—The theory of structural controllability allows us to assess controllability of a network as a function of its interconnection graph and independently of the edge weights. Yet, existing structural controllability results require the weights to be selected arbitrarily and independently from one another, and provide no guarantees when these conditions are not satisfied. In this note we develop a new theory for structural controllability of networks with symmetric, thus constrained, weights. First, we show that network controllability remains a *generic* property even when the weights are symmetric. Then, we characterize necessary and sufficient graph-theoretic conditions for structural controllability of networks with symmetric weights: a symmetric network is structurally controllable if and only if it is structurally controllable without weight constraints. Finally, we use our results to assess structural controllability from one region of a class of empirically-reconstructed brain networks.

Index Terms—Network controllability; structural controllability; interconnected systems; graph theory; symmetric networks.

I. INTRODUCTION

The question of controllability of complex network systems arising in engineering, social, and biological domains has been the subject of intensive study in the last few years [1]–[3]. One key question motivating the investigation is to characterize relationships and tradeoffs between the interconnection structure of a network and its controllability [4]–[6]. To this end, graphical tools from structural systems theory [7]–[10] are typically preferred over algebraic controllability tests, which suffer from numerical instabilities when the network cardinality grows, require exact knowledge of the network weights, and are agnostic to the graph supporting the dynamics.

While the theory of structured systems and generic properties of linear systems is well-developed and understood [11], all results assume the possibility of assigning the network weights arbitrarily and independently from one another. In fact, when this condition is violated, the conclusions drawn from structural analysis may lead to incorrect results [7], [12]. Unfortunately, it is often the case that this assumption is violated in real networks due to physical, technological, or biological reasons. For instance, the small-signal network-preserving model of a power network contains a Laplacian submatrix, whose entries are symmetric and satisfy linear constraints (row sums equal to zero) [13], [14]. Similar constraints appear also when studying synchronization in networks of

Kuramoto oscillators [15] and general systems with consensus dynamics [16]. Novel theories and tools are needed to study controllability of networks with constrained weights.

In this paper we focus on networks with symmetric weights and derive graph-theoretic conditions for their structural controllability from dedicated control inputs. While (group) symmetry has previously been found to be responsible for network uncontrollability [17], [18], the question of how symmetric edge weights affect structural controllability has not been investigated, with the exception of [19]. In [19], however, the proposed conditions for structural controllability of undirected (symmetric) networks are implicit and based on the generalized zero forcing sets to estimate the dimension of the controllable subspace. Similarly, although the recent paper [20] studies structural controllability for a class of networks with constrained parameters, this class of network matrices does not contain the set of symmetric matrices considered in this work. Thus, the necessary and sufficient conditions derived in this paper are the first graph-theoretic conditions for structural controllability of networks with symmetric weights.

The contribution of this paper is three-fold. First, we show that controllability of symmetric networks is a generic property. That is, either the network is controllable for almost all symmetric choices of interconnection weights, or it is not controllable for all symmetric weights. This first result can be easily extended to different classes of constraints other than symmetry. Second, we show that a network with symmetric weights is structurally controllable if and only if it is spanned by a (symmetric) cactus rooted at the control node. By comparing our result with those in [21], our analysis shows that a network is structurally controllable with symmetric weights if and only if it is structurally controllable with unconstrained weights. Third, we use our results to show that a class of (symmetric) brain networks reconstructed from diffusion MRI data is structurally controllable from a single dedicated control region. Finally, we note that, due to duality between controllability and observability, the results of this paper extend directly to the study of structural observability of networks with symmetric weights and a dedicated sensor.

The rest of the paper is organized as follows. Section II contains our network model and preliminary notions. Section III contains our analysis and conditions for structural controllability of networks with symmetric weights, and some examples. Finally, Section IV contains an illustrative example featuring brain networks, and Section V concludes the paper.

II. PROBLEM SETUP AND PRELIMINARY NOTIONS

We study controllability of symmetric network systems, which are described by a weighted directed graph (digraph)

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