On Cooperative Patrolling: Optimal Trajectories, Complexity Analysis, and Approximation Algorithms

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***Abstract*—The subject of this paper is the patrolling of an environment with the aid of a team of autonomous agents. We consider both the design of open-loop trajectories with optimal properties and of distributed control laws converging to optimal trajectories. As performance criteria, the *refresh time* and the *latency* are considered, i.e., respectively, time gap between any two visits of the same region and the time necessary to inform every agent about an event occurred in the environment. We associate a graph with the environment, and we study separately the case of a chain, tree, and cyclic graph. For the case of chain graph, we first describe a minimum refresh time and latency team trajectory and propose a polynomial time algorithm for its computation. Then, we describe a distributed procedure that steers the robots toward an optimal trajectory. For the case of tree graph, a polynomial time algorithm is developed for the minimum refresh time problem, under the technical assumption of a constant number of robots involved in the patrolling task. Finally, we show that the design of a minimum refresh time trajectory for a cyclic graph is *NP-hard*, and we develop a constant factor approximation algorithm.**

***Index Terms*—Algorithm design and analysis, combinatorial mathematics, distributed algorithms, graph partition, mobile agents, multirobot systems, network theory (graphs), patrol.**

I. Introduction

The recent development in the autonomy and the capabilities of mobile robots greatly increases the number of applications that are suitable for a team of autonomous agents. Particular interest has been received by those tasks requiring continual execution, such as the monitoring of oil spills [1], the detection of forest fires [2], the track of border changes [3], and the patrol (surveillance) of an environment [4]. The surveillance of an area of interest requires the robots to continuously and repeatedly travel the environment, and the challenging problem consists in scheduling the robots trajectories to optimize a certain performance criteria. The reader familiar with network location, multiple traveling salesman, or graph exploration problems may observe a close connection with the patrolling problem we address (see, e.g., [5]–[7]). It is worth noting, however, that these classical optimization problems do not capture the repetitive, and hence dynamic, aspect of the patrolling problem, nor the synchronization issues that arise when a timing among the visits of certain zones is required.

A precise formulation of the patrolling problem requires the characterization of the robots capabilities, of the environment to be patrolled, and of the performance criteria. In this paper, we assume the robots to be identical and capable of sensing and communicating within a certain spatial range, and of moving according to a first-order integrator dynamics with bounded speed. We represent the environment as a graph, in which the vertices correspond to physical and strategically important locations, and the edges denote the possibility of moving and communicating between locations. We assume that, when a robot is placed at each of the graph vertices, the union of the sensor footprints provides complete sensor coverage of the environment. Regarding the performance criteria of a patrolling trajectory, we consider 1) the time gap between any two visits of the same region, which is called *refresh time*, and 2) the time needed to inform the team of robots about an event occurred in the environment, which is called *latency*. Loosely speaking, refresh time and latency reflect the effectiveness of a patrolling team in detecting events in the environment and in organizing remedial actions. For both the refresh time and latency optimization problem, we focus on the worst case analysis, even though the average refresh time and the average latency cases are also of interest. Notice that for the latency to be finite, the motion of the robots needs to be synchronized. For instance, if two robots are allowed to communicate only when they simultaneously occupy two adjacent vertices of the graph, then they need to visit those vertices at the same time in a finite latency trajectory.

The patrolling problem is receiving increasing attention because of its fundamental importance in many security applications (see, e.g., [8]–[12]). Although many solutions have been proposed, the problem of designing minimum refresh time and latency team trajectories for a general environment is, to date, an open problem. Almost all traditional approaches rely on space decomposition and traveling salesperson tour computation [13]. In [14], an empirical evaluation of existing patrolling heuristics is performed. In [15], two classes of strategies are presented, namely, the cyclic-based strategy and the partition-based strategy. In the cyclic-based strategy, the robots compute a closed route through the viewpoints and travel repeatedly such route at maximum speed. Clearly, in the case of a single robot, if the tour is the shortest possible, then the cyclic-based strategy performs optimally with respect to the refresh time and latency criteria. In the partition-based strategy, the viewpoints are partitioned into m subsets, m being cardinality of the team, and

Manuscript received June 6, 2011; revised September 22, 2011; accepted November 28, 2011. Date of publication January 4, 2012; date of current version June 1, 2012. This paper was recommended for publication by Associate Editor N. Y. Chong and Editor J.-P. Laumond upon evaluation of the reviewers' comments. This work was supported in part by the National Science Foundation under Grant IIS-0904501 and Grant CPS-1035917.

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Digital Object Identifier 10.1109/TRO.2011.2179580